Maximum entropy and Goertzel's methods of spectral analysis for tsunami waves

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Maximum Entropy and Goertzel's Methods of Spectral Analysis for Tsunami Waves

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Abstract

Two methods of spectral analysis, one of which is maximum entropy method and another is Goertzel's method, were applied to the tsunami waves and compared.

As a result it is suggested that the former is better to identify the locations of peaks and the later is better to obtain the relative amplitudes among many spectral peaks. Tsunami tide gauge records at six stations, obtained in the neighborhood of Niigata City, Japan on May 26, 1983, were analized and it is shown that the tsunami included the period components of 41-46 min as the largest ones on the maximum entropy method. It is described that these components are characteristic to the tsunami and the small differences among the periods at these stations were attributed to modification due to the local topographies.

Introduction

Periodic properties of tsunami waves are important to study the excitation mechanism. Spectral analysis is one of the methods to obtain the periodicity. The spectral analysis is consisted of two groups. One of the direct method and another is an indirect method. In the later the indirectness originates in an indirect calculation through auto-correlation coefficients. A recent development of spectral analysis is combined with an increase of memories and operation velocity of computers. In direct methods, FFT (Fast Fourier Transform) is frequently used but it is under a restriction of the same coefficients as the sampling points. On the other hand Goertzel's method, (Goertzel, 1958) being free from such a restriction, was noticed by Gentleman (1969). It is easy to apply this method to the wave analysis because of a simple algorithm. Abe and Ishii (1983) applied this method to Pacific tsunamis and revealed shelf oscillations. In the indirect method, recently a maximum entropy method has been noticed. At first Burg (1967) proposed it and Akaike (1969) presented another one equivalent to the former. The later one assumes that a stochastic process is represented in the use of an auto-regressive model and leads a spectrum, being satisfied with the maximum entropy relation, from the auto-regressive model, of which coefficients are determined with auto-correlation. Sanchez and Farreras (1983) used the maximum entropy method to analyze tsunami waves in the first time and pointed out the excitation of natural oscillations due to tsunamis at Mexican Coast.

It is expected that an application of both the methods to the same tsunami clarifies an advantage and disadvantage. In this paper we apply them to the tsunami tide gauge records obtained on May 26, 1983 and compare the result.

Maximum entropy and Goertzel's methods

(i) Maximum entropy method

Time series x $(=x(k\Delta t))$, in which Δt is a sampling interval and k is an integer, is represented by Akaike (1969) with an auto-regressive model as

$$x_k = \sum_{i=1}^{m} a_{mi} x_{k-i} + n_k \tag{1}$$

in which a_{mi} is an auto-regressive coefficient and n_k is a white noise. We use a notation by Minami (1986). On the other hand an auto-correlation coefficients R_i is defined as follows:

$$R_i = R(i\Delta k) = E\{x_k \ x_{k-i}\} + E\{x_k \ n_k\} \tag{2}$$

in which $E\{\ \}$ means an expectation value. Combining (1) with (2) we obtain an expression of

$$R_0 = E\{x_k^2\} = -\sum_{i=1}^{m} a_{mi} E\{x_k x_{k-i}\} + E\{x_k \; n_k\}$$
$$= -\sum_{i=1}^{m} a_{mi} R_i + E\{n_k^2\} \tag{3}$$

In this expression an independence of n_k with x_k is assumed. Taking expectation values on the multiplication of $x_{k-1}, x_{k-2}, \cdots, x_{k-m}$ to the formula (1), we can get a matrix representation of

$$\begin{bmatrix} R_0 R_1 \cdots R_m \\ R_1 R_0 \cdots R_{m-1} \\ \cdots\cdots\cdots\cdots\cdots \\ R_m R_{m-1} \cdots R_0 \end{bmatrix} \begin{bmatrix} 1 \\ a_{m1} \\ \cdot \\ a_{mm} \end{bmatrix} = \begin{bmatrix} P_m \\ 0 \\ \cdot \\ 0 \end{bmatrix} \tag{4}$$

in which $P_m = E\{n_k^2\} = \sigma^2$ is equal to the average power of m point prediction error filter. Solving the equation we obtain unknown parameters of $P_m, a_{m1}, a_{m2} \cdots a_{mm}$. Power spectral density is calculated using the expression of

$$S(\omega) = \frac{P_m \Delta t}{|1 + \sum_{i=1}^{m} a_{mi} e^{-j\omega i \Delta t}|^2} \tag{5}$$

in which j is an imaginary number and ω is an angular frequency.

(ii) Goertzel's method

Fourier coefficients in finite discrete time series x_k $(k=0 \sim n-1)$ are defined as

$$\left.\begin{aligned} a(\omega) &= \sum_{k=0}^{N-1} x_k \cos k\omega \\ b(\omega) &= \sum_{k=0}^{N-1} x_k \sin k\omega \end{aligned}\right\} \tag{6}$$

In Goertzel's method $a(\omega)$ and $b(\omega)$ are obtained (e.g. Gentleman, 1969) with formula of

$$\left.\begin{aligned} a(\omega) &= x_0 + u_1 \cos \omega - u_2 \\ b(\omega) &= \qquad u_1 \sin \omega \end{aligned}\right\} \tag{7}$$

in which

$$\left.\begin{aligned} u_N &= u_{N+1} = 0 \\ u_j &= x_j + 2\cos \omega \cdot u_{j+1} - u_{j+2} \end{aligned}\right\} \tag{8}$$

This formalism is characterized with only one calculation of trigonometric function and a simple recurrence relation. This gives a spectral amplitude.

Preliminary application

Preceding the application to the tsunami tide gauge records we applied both the methods to a psued tsunami consisting of two sine curves, having the periods of 20 and 60 min, as

$$x_k = 2 + \sin(0.1(k+10)\pi\Delta t) + \sin(0.0333(k+10)\pi\Delta t) \quad (9)$$

in which time interval Δt is selected to be 2 minutes. In the calculation using a micro computer, Minami's program (1986) was used for the maximum entropy method (MEM) and the number of coefficients of auto-regressive model is fixed to be 30. Total sampling time is 12 hours. For this calculation the result is shown in Fig. 1. A power spectral density are calculated in MEM and spectral amplitude are calculated in Goertzel's method. Common unit in the ordinate is ineffective to represent the spectral peaks of both the cases. Thus we conclude that MEM gives correct locations of spectral peaks and incorrect amplitude of spectra. On the other hand Goertzel's method gives a correct spectral amplitude and peak location, but the broadened peak bring a difficulty to read an exact location of the peak.

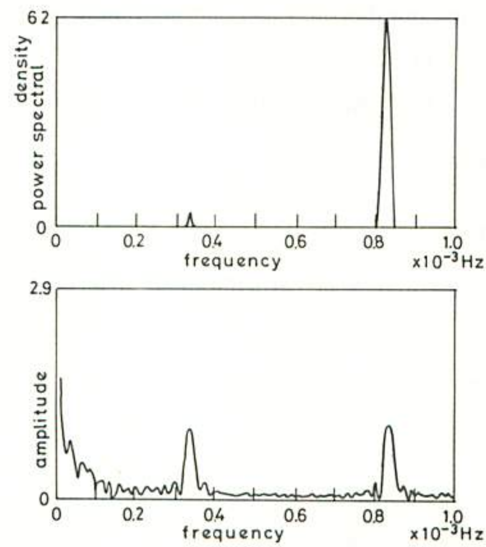

Fig. 1 Comparison of power spectral density obtained on the maximum entropy method (Upper figure) with spectral amplitude obtained on Goertzel's method (Lower figure) for the tsunami consisting of two sine curves.

Spectral analysis

Tsunami tide gauge records, obtained at six stations in the neighborhood of Niigata City on May 26, 1983, were analyzed. The records were digitized by the use of digitizer in an arbitrary time interval from 12 h 20 m to 0 h 20 m (May 27) and the time series was rearranged with a constant time interval of 2 minutes. The spectra of the constant interval time series were calculated. We had no preliminary data processing such as filtering. The result of the maximum entropy method is shown in Fig. 2.

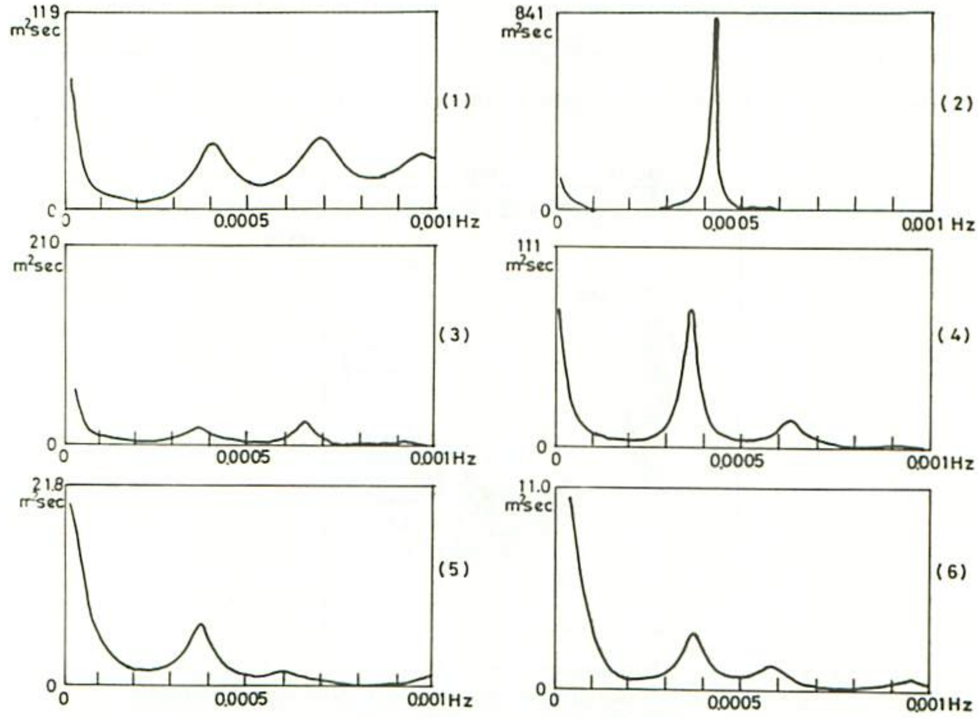

Fig. 2 Power spectral densities calculated on the maximum entropy method to the tsunami tide gauge records at six stations (1) Iwafune, (2) Niigata East Port, (3) Agano River, (4) Niigata West Port, (5) Nishikawa Gate, (6) Teiseki Bridge.

In the maximum entropy method the spectra show simple forms with no short-perioded variations. Thus it is easy to point out the location of the spectral peaks. Highly resoluted spectral peaks are obtained. It is shown in Fig. 2 that the peak period at Niigata East Port was the shortest at 0.000410 Hz (=40.7 min) and one at Niigata

West Port has the second sharpest peak at 0.000367 Hz (=45 min). These two ports have the breakwaters against waves from the west and have the mouthes facing to the north as shown in Fig. 3. The tsunami directs the incoming direction to the north in the neighborhood of Niigata City. It is observed that the direction of mouthes is favorable to receive the wave from the north, having the period componentof about 40 minutes.

The stations of Nishikawa Gate and Teiseki Bridge are located 9 and 10 km distant from the mouth of Shinano River, respectively. Both the points show the same peak frequency of 0.000380 Hz (=43.9 min) with small power amplitude. The river proved to lead the incoming wave of 40 minuted at a long distance. The largest predominant periods, calculated on the maxinum entropy method, are shown on the map in Fig. 3. This figure shows a kind of block structure of the predominant period. This fact suggests a modification of the incoming wave resulting by the local topographies.

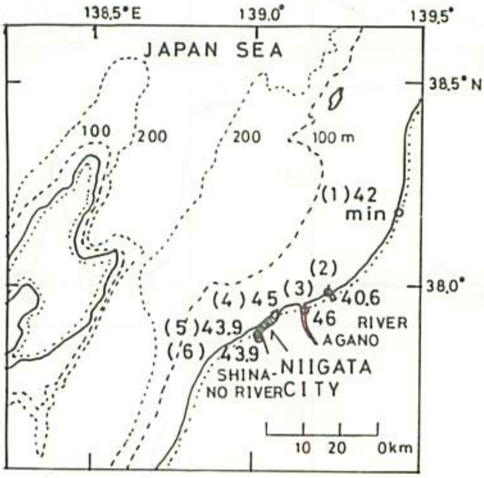

Fig. 3 Location of tide gauge stations and the largest predominant periods observed in the tsunami on May 26, 1983 (Numbers in parentheses correspond with the station numbers in Fig. 2).

The result on Geortzel's method is shown in Fig. 4. In Goertzel's method the spectral curve includes a short-period fluctuation. This fluctuation decreases a resolution of peak frequency. Since we were afraid that the direct current component of the reading data prevented from reproducing a correct peak, reduction of direct current component from the reading data was tried through replacing the reading amplitude and averaged one but the direct components were not reduced from the result. Thus we found a reason of no using of prefiltration. It was also affirmed that replotting of power density spectral amplitude in the ordinate did not succeed in emphasizing the spectral peaks. An important fact is that the spectral peaks obtained on Goertzel's methods coincide with ones obtained with maximum entropy method and the spectral amplitude on the former method is almost proportional with the power spectral density on the later method in log-log plotting as shown in Fig. 5. In the figure the power spectral density on Goertzel's method is defined as the multiplicathons of amplitude squared by the sampling interval (=120 sec).

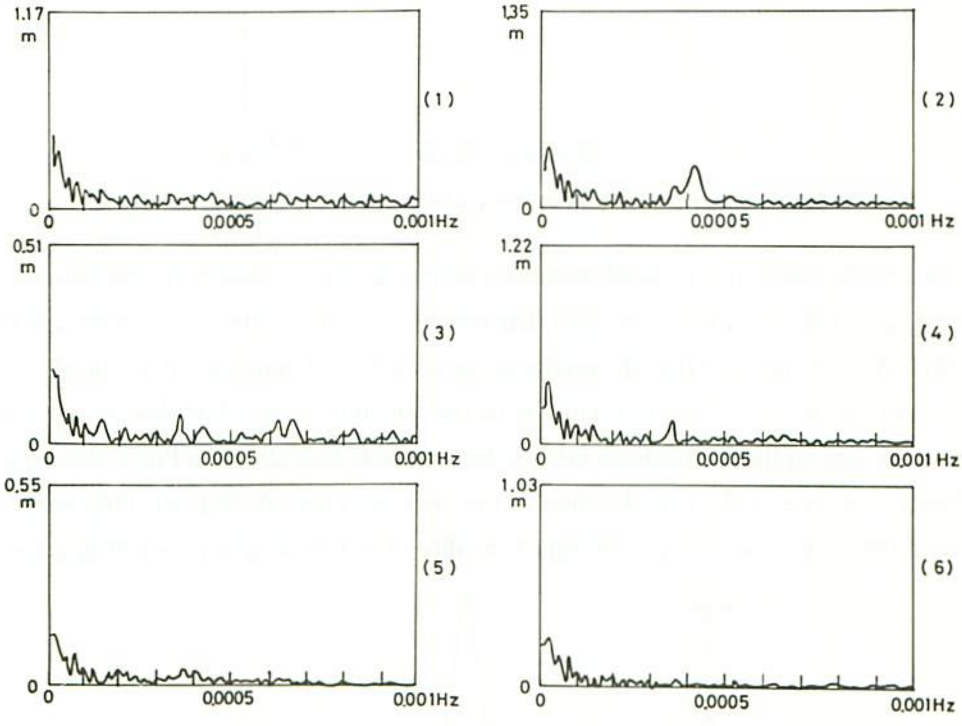

Fig. 4 Spectral amplitude calculated on Goertzel's method to the tsunami tide gauge records at six stations (the same stations as those in Fig. 2).

Finally we show that the predominant period of 40 minutes, which were observed at all stations, were characteristic to Japan Sea Tsunami on May 26, 1983 in comparison with Niigata Tsunami on June 16, 1964. Fig. 6 is the power spectral density calculated with the maximum entropy method for the tide gauge record obtained at Agano River. This figure shows the most predominant period to be 30.2 minutes and the

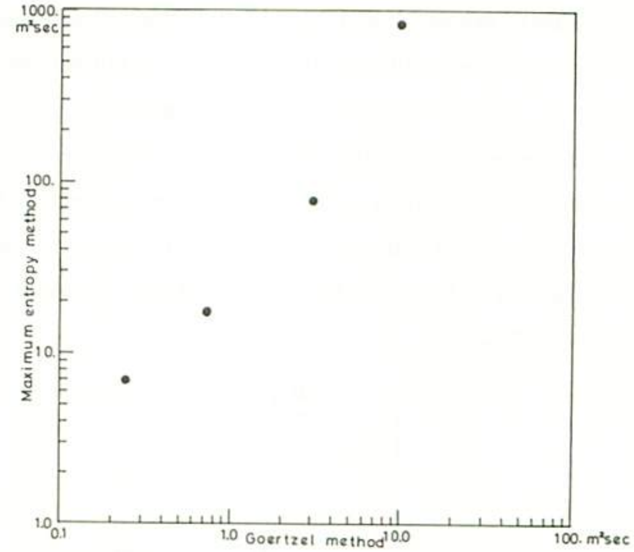

Fig. 5 Comparinon of power spectral densities on both the methods.

second predominant one to be 64 minutes. Comparing these values with ones obtained at the same station in Fig. 2, we find that these values are shifted to a longer side of period. We can conclude that the predominant period of 40 minutes is not related with the observation station but is related with the tsunami source. We observe a small difference among the predominant periods for six stations as shown in Fig. 5. Obtaining a block structure of the period values in the map, we conclude that the difference is caused from the local topography, which modifies the incoming wave of 40 minutes.

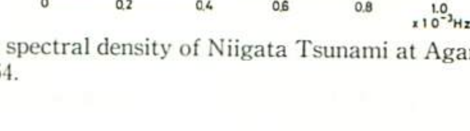

Fig. 6 Power spectral density of Niigata Tsunami at Agano River on June 16, 1964.

Discussion and conclusion

Ulrych, T. J. (1972) and Radosky, H. R. *et al.* (1975) compared the effectiveness of determining the spectral peak location by means of the maximum entropy with one by auto-correlation method, applying these methods to a curve of a few sinusoids. They showed the maximum entropy method to be highly resolutive to the spectral peak. We compared the maximum entropy method with Goertzel's method and showed that the maximum entropy method has a resolutive power higher than Goertzel's method. Applying the maximum entropy method to tsunami waves observed in the neighborhood of Niigata City on May 26, 1983, we found a predominant period of about 40 minutes in the incomming waves. This discovery was not brought about by means of Goertzel's method. Thus we conclude that we must use the maximum entropy method to discuss fine spectra of tsunami waves.

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